Insertion Devices-1

Friday 28 July, 2006 in YangZhou, 10:15–12:15

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Introduction

- Insertion devices include the wigglers and undulators that are magnetic devices producing a specially periodic field variation.
- They are all placed in the straight sections of storage ring.
- Wiggler spectrum at higher photon energies is smooth, similar to that of a bending magnet. However, the radiation intensity can be much higher as much as increased numbers of poles and higher magnetic field which generate radiation with a higher critical energy.
- When the use of periodic magnets in a regime in which interference effects is coherent, and then the device is called “undulator”.
- The main radiation features of insertion devices are (1) higher photon energy, (2) higher flux and brightness, (3) different polarization characteristics.
- The first undulators to be installed in storage rings were at the VEPP-3 ring at INP, Novosibirsk in 1979.
- Superconducting wigglers are currently operating in several synchrotron radiation facilities: SRS (England), DCI and ESRF (France), UVSOR and Photon Factory (Japan) and NSLS Xray Ring and CAMAD (USA), SRRC (Taiwan), BASSY II.
- Superconducting Undulator: Just in development like as ANKA (Germany) superconducting in-vacuum undulator.
Fig. 1 reveals that an electron beam traveling in a curved path (Bending magnet) at nearly the speed of light emits photons into a narrow cone of natural emission angle $\cong \gamma^{-1}$. Fig. 2 shows that Electrons beam traveling wiggler or undulate in the midplane along the spatially periodic sinusoidal field in an insertion device at nearly the speed of light emits photons into a narrow cone of natural emission angle $\cong k\gamma^{-1}$. Where, $k$ is the deflection parameter. The radiation pattern from different type of magnets was compared in the Fig. 3. For the dipole magnet, there is a wide band synchrotron radiation in horizontal plane and a narrow cone in vertical. For the wiggler, the horizontal radiation cone become is $k\gamma^{-1}$ and the vertical cone is the same as that of the dipole magnet. However, for the undulator, the radiation cone in horizontal and vertical are all closed to be $\gamma^{-1}$. Here, we have defined the deflection parameter of $k$ is closed to 1.

Fig. 1. The synchrotron radiation emitted from an electron beam which was bent in a curved path.
Basic features of the radiation from standard insertion devices

Fig. 2. The synchrotron radiation emitted from an electron beam which was bent in a spatially periodic sinusoidal field in an insertion device.

Fig. 3. The synchrotron radiation emitted from (a) bending magnet, (b) wiggler, (c) undulator.
Spectrum difference between wiggler & undulator
Mechanism for various polarization mode

\[ B_{xm}, B_{ym}, \text{and } B_{zm} \text{ are the maximum field strength in the } x, y, \text{and } z \text{ directions which values depend only on the magnet gap, } g_m. \ \phi_A, \ \phi_B, \ \phi_C, \text{and } \phi_D \text{ are the phase-shifts of rows } A, B, C \text{ and } D, \text{which are defined as } \frac{2\pi z_A}{\lambda u}, \ \frac{2\pi z_B}{\lambda u}, \ \frac{2\pi z_C}{\lambda u}, \text{and } \frac{2\pi z_D}{\lambda u}, \text{respectively.} \]

\[ B_x = B_{xm} \left[ -\cos(\phi_A) + \cos(\phi_B) + \cos(\phi_C) - \cos(\phi_D) \right]/4 \]

\[ B_y = B_{ym} \left[ \cos(\phi_A) + \cos(\phi_B) + \cos(\phi_C) + \cos(\phi_D) \right]/4 \]

\[ B_z = B_{zm} \left[ \sin(\phi_A) + \sin(\phi_B) - \sin(\phi_C) - \sin(\phi_D) \right]/4 \]
Elliptically Polarized Undulator

**Horizontal linear polarization**

\[ B_x = 0, \quad B_y = 4B_{ym} \sin(2\pi z/\lambda_u) \quad z_A = z_B = z_C = z_D = z_O = 0 \]

**Circular polarization**

\[ B_x = -4B_{xm} \sin(\pi/2 + (2\pi z/\lambda_u) + (\pi z_0/\lambda_u)) \sin(\pi/2 + (\pi z_0/\lambda_u)), \quad B_y = 4B_{ym} \sin(2\pi z/\lambda_u + (\pi z_0/\lambda_u)) \sin(\pi z_0/\lambda_u) \quad z_B = z_C = z_0 = \lambda_u/2 \]

**Vertical linear polarization**

\[ B_x = 4B_{xm} \sin(2\pi z/\lambda_u), \quad B_y = 0 \quad z_B = z_C = z_0 = \lambda_u/2 \]

**Linear polarization in any direction**

\[ B_x = 2B_{xm} [\sin(2\pi z/\lambda_u) - \sin(2\pi z/\lambda_u) \cos(2\pi z_0/\lambda_u)] - z_B = z_C = \lambda_u/2 \]

\[ B_y = 2B_{ym} [\sin(2\pi z/\lambda_u) + \sin(2\pi z/\lambda_u) \cos(2\pi z_0/\lambda_u)] \]
Adjustable phase undulator APU10

By = Byo cos(\(\frac{\Delta z}{2}\))

Bz = ByoF sin(\(\frac{\Delta z}{2}\))

\(\theta\): phase shift (\(2\pi\Delta z/\lambda\))

\(B = By\) at \(\Delta z = 0\)

\(B = Bz\) at \(\Delta z = \pi\)

F: Structure factor

A array

B array
Field features of APU10

\[ \frac{d^2 y}{dz^2} = -\frac{1}{2} \left( \frac{eB_{HAPU}}{E} \right)^2 y = -k_y y \]

\[ \frac{d^2 y}{dz^2} = -\frac{1}{2} \left( \frac{eB_{HAGU}}{E} \right)^2 y = -k_y y \]

\[ \Delta Q_y = \frac{k_y \beta_y L}{4\pi} \left( 1 + \frac{L^2}{12 \beta_y^2} \right) \]

\[ \Delta \beta_y = \frac{k_y \beta_y^2 L}{2 \sin \mu} \left( 1 - \frac{L^2}{12 \beta_y^2} \right) \]
Electron motion

\[ B_y = B_0 \sin(kz) \], where \( k = \frac{2\pi}{\lambda_0} \) and \( \lambda_0 \) is the insertion device period length.

\[ \dot{x} = \frac{e}{\gamma m} (-\dot{z}B_y) \quad \dot{z} = \frac{e}{\gamma m} (\dot{x}B_y) \]

\[ \dot{x} = \frac{eB_o \cos(kz)}{\gamma m k} \quad \beta_x = \dot{x} / c = \frac{K}{\gamma} \cos(kz) \]

where the dimensionless undulator or deflection parameter is defined as follows:

\[ K = \frac{eB_o\lambda_0}{2\pi mc} = 0.934B_o[T\lambda_o][cm] \]

\[ \beta^2_x + \beta^2_z = \beta^2 \quad (= \text{constant}) \]

\[ \beta_z \approx \beta \left( 1 - \frac{K^2}{4\gamma^2} - \frac{K^2}{4\gamma^2} \cos 2kz \right) \]

The average velocity along the z-axis is thus: \( \bar{\beta} \approx \beta \left( 1 - \frac{K^2}{4\gamma^2} \right) \approx 1 - \frac{1}{2\gamma^2} - \frac{K^2}{4\gamma^2} \)

We will only consider cases in which \( K/\gamma << 1 \) and so we can write to a good approximation that \( z = \bar{\beta}ct \) and \( kz = \Omega t \) where \( \Omega = 2\pi\bar{\beta}c / \lambda_o \). We have electron angle then:

\[ \dot{x} = \frac{K}{\gamma} c \cos(\Omega t) \quad \dot{z} = \bar{\beta}c - \frac{K^2}{4\gamma^2} c \cos(2\Omega t) \quad \dot{x}' = \frac{K}{\gamma} \cos(\Omega t) \]

which can be integrated directly to give e-trajectory: \( x = \frac{K}{\gamma} \frac{c}{\Omega} \sin(\Omega t) \quad z = \bar{\beta}ct - \frac{K^2}{4\gamma^2} \frac{5910}{2\Omega} \sin(2\Omega t) \)
Interference

In the time it takes the electron to move through one period length from point A to an equivalent point B ($\lambda_o / \beta c$) the wavefront from A has advanced by a distance $\lambda_o / \beta$ and hence is ahead of the radiation emitted at point B by a distance $d$ where:

$$d = \frac{\lambda_o}{\beta} - \lambda_o \cos \theta$$

and where $\theta$ is the angle of emission with respect to the electron beam axis. When this distance is equal to an integral number, $n$, of radiation wavelength there is constructive interference of the radiation from successive poles:

$$\frac{\lambda_o}{\beta} - \lambda_o \cos \theta = n\lambda$$

Inserting the expression for the average electron velocity:

$$\frac{1}{\beta} \approx 1 + \frac{1}{2\gamma^2} + \frac{K^2}{4\gamma^2}$$

results in the following interference condition:

$$\varepsilon [\text{keV}] = 0.95n \frac{E^2 [\text{GeV}]}{\lambda_o \left(1 + \frac{K^2}{2} + \gamma^2 (\theta^2 + \psi^2)\right)}$$

$K >> 1$ wiggler

$K \approx 1$ undulator
Radiation from bending & wiggler magnet

In a wiggler, the deflection parameter $K$ is large (typically $K \geq 10$) and photon radiation from different poles of the electron trajectory is enhanced incoherently. The angular density of flux is then given by $2N$ ($N$ is the number of magnet periods) times the formula for bending magnets. The angular distribution of radiation emitted by electrons that are moving through a bending magnet, following a circular trajectory in a horizontal plane is,

$$
\frac{d^2B(w)}{d\theta d\varphi} = \frac{3\alpha \gamma^2}{4\pi^2 e} \frac{I}{\Delta w} \left( \frac{\varepsilon}{\varepsilon_c} \right)^2 \left( 1 + \gamma^2 \varphi^2 \right)^2 \left[ K_{2/3}(\xi) + \frac{\gamma^2 \psi^2}{1 + \gamma^2 \psi^2} K_{1/3}(\xi) \right]
$$

Where $\varepsilon$ and $\varepsilon_c$ are the photon energy and the photon critical energy, respectively; $\theta$ and $\varphi$ are the observation angles in the horizontal and vertical directions, respectively; $\alpha$ is the fine-structure constant; $I$ is the beam current; $e$ is the electron charge; the subscripted $K$’s are modified Bessel functions of the second kind, and $\xi$ is defined as

$$
\xi \equiv \left( \varepsilon / 2 \varepsilon_c \right) \left( 1 + \gamma^2 \psi^2 \right)^{3/2}
$$

$$
\varepsilon_c(\theta) = \varepsilon_c(0) \sqrt{1 - \left( \theta \gamma / K \right)^2}
$$
Radiation from insertion devices

**Spectral/angular distribution**

\[
\frac{d^2 I}{d\omega d\Omega} = \frac{c}{4\pi^2} \left| \int_{-\infty}^{\infty} R E(t) e^{i\omega t} \, dt \right|^2
\]

\[
E(t) = \frac{e}{\sqrt{4\pi\epsilon_0 c}} \left[ \hat{n} \wedge \left( (\hat{n} - \beta) \wedge \beta \right) \right]_{t_{\text{ref}}}
\]

where \( \hat{n} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \) is the unit vector from the point of emission to the observer (see Figure). The observer and emission times are related by:

\[
t = t_{\text{ref}} + R/c\]

where \( R \) is the distance between the emission and observer points, and hence:

\[
\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{(4\pi\epsilon_0)^2 \pi^2 c} \left| \int_{-\infty}^{\infty} \left( \hat{n} \wedge \left( (\hat{n} - \beta) \wedge \beta \right) \right) e^{i\omega(t-R/c)} \, dt \right|^2
\]

\[
\frac{d^2 I}{d\omega d\Omega} = \frac{e^2 \gamma^2 N^2}{(4\pi\epsilon_0)^2 c} L(N\Delta \omega / \omega_1(\theta)) F_n(K, \theta, \phi)
\]

Geometry for the analysis of undulator radiation
Radiation from undulator

Angular flux density function in the horizontal (left) and vertical (right) planes for the case $K = 1$

$$F_n(K) = \frac{n^2 K^2}{(1 + K^2 / 2)^2} \left[ J_{\frac{n+1}{2}}(Z) - J_{\frac{n-1}{2}}(Z) \right]^2$$

$$Z = \frac{nK^2}{4(1 + K^2 / 2)}$$
Radiation from undulator

On-axis angular flux density function

in practical units of photons/s/mrad$^2$/0.1% bandwidth:

\[
\bar{\beta} = \left. \frac{d^2 \dot{n}}{d\omega / \omega \cdot d\Omega} \right|_{\theta=0} = 1.744 \cdot 10^{14} N^2 E^2 [GeV] F_n(K) I_b
\]
Radiation from undulator

◆ Total flux

We obtain therefore in practical units of flux is photons/s/0.1% bandwidth:

\[ B = B \frac{d\Omega}{d \omega} = \frac{d\bar{n}}{d \omega} = 1.431 \times 10^{14} NQ_n(K) f_n(N\Delta \omega / \omega_n(0))I_b \]

where \( Q_n(K) = \left( 1 + K^2 / 2 \right) F_n(K) / n \). The flux function \( Q_n(K) \) and the detuning function \( f(N\Delta \omega / \omega_n(0)) \). It can be seen that for zero detuning (i.e. \( \omega = \omega_n(0) \)) the flux is very close to half of the usually quoted result. Nearly twice as much flux can be obtained however by a small detuning to lower frequency by approximately \( \Delta \omega / \omega_n(0) \approx -1/N \)

Undulator flux function (left) and flux as a function of detuning (right)
Radiation from insertion devices

◆ Brightness

We will obtain the brightness in practical units is photon/s/mm²/mrad²/0.1% bandwidth. Photon flux unit is photon/s/0.1% bandwidth.

\[
\beta = \frac{B}{(2\pi)^2 \cdot \sigma_{Tx} \cdot \sigma_{Ty} \cdot \sigma_{Tx'} \cdot \sigma_{Ty'}}
\]

\[
\sigma_{Tx} = \sqrt{\sigma_x^2 + \sigma_p^2} \quad \quad \sigma_{Ty} = \sqrt{\sigma_y^2 + \sigma_p^2}
\]

\[
\sigma_{Tx'} = \sqrt{\sigma_{x'}^2 + \sigma_{p'}^2} \quad \quad \sigma_{Ty'} = \sqrt{\sigma_{y'}^2 + \sigma_{p'}^2}
\]

The quantity \( \varepsilon_x = \sigma_x \sigma_{x'} \) and \( \varepsilon_y = \sigma_y \sigma_{y'} \). The diffraction-limited source size (rms) corresponding to the angular divergence \( \sigma_p \) is \( \sigma_p \cdot \sigma_{p'} = \frac{\lambda_p}{4\pi} \) and \( \sigma_p = \frac{1}{4\pi} \sqrt{\lambda_u L} \).

\( \lambda_p \) is the photon wavelength and \( \lambda_u \) is the undulator periodic length.
Radiation from insertion devices

◆ Power and power density

\[
\frac{dP}{d\Omega} [W \text{ / mrad}^2] = 10.84E^4[GeV]B_0NI_bG(K)f_K(\theta_x, \theta_y)
\]

where \( G(K) = \frac{K(K^6 + \frac{24}{7}K^4 + 4K^2 + \frac{16}{7})}{(1 + K^2)^{\frac{3}{2}}} \) and

\[
f_K(\theta_x, \theta_y) = \frac{16K}{7\pi G(K)} \int_{-\pi}^{\pi} \sin^2 \alpha \left[ \frac{1}{D^3} - \frac{4(\theta_x - K \cos \alpha)^2}{D^5} \right] d\alpha \text{, as obtained by Kim.}
\]

\[
P_{tot}[W] = 633 \cdot E[GeV]B_0^2LI_b
\]
Summarized of the radiation features of bending and insertion devices magnet

- Bending magnet
- Wiggler
- Undulator
Bending magnet

Brightness:

\[
\beta = \frac{B}{2\pi} \left[ \sigma_x^2 + D^2 \sigma_e^2 + \sigma_r^2 \right] \left[ \sigma_y^2 + \sigma_r^2 + \frac{\varepsilon_y^2 + \gamma_y \sigma_r^2}{\sigma_\psi^2} \right]^{\frac{1}{2}}
\]

where \( \gamma_y \) is the third betatron function in the y direction, \( \varepsilon_y \) is the beam emittance, \( D \) is the dispersion function in x direction, \( \sigma_e \) is the energy spread, \( \sigma_\phi \) is the angular width-effective rms half-angle.

The critical energy and total power of Bending magnet radiation is

\[
E_c[keV] = 0.665 E^2[GeV] B[T]
\]

\[
\sigma_r = \frac{\lambda}{(4\pi \sigma_\psi)}
\]

\( \lambda \) is the photon wavelength; \( \sigma_\psi = 0.65/\gamma \) at critical energy.
Wiggler

Brightness:

\[
\beta = \frac{\bar{B}}{\sum_{\pm} \sum_{n=\left[-\frac{N}{2}\right]}^{\left\lfloor \frac{N}{2} \right\rfloor} \frac{1}{2\pi} \exp \left[ -\frac{1}{2} \left( \frac{x_0^2}{\sigma_x^2 + Z_{n\pm}^2 \sigma_{x'}^2} \right) \right] \left( \sigma_x^2 + Z_{n\pm}^2 \sigma_{x'}^2 \right) \left( \frac{\varepsilon_y^2}{\sigma_{\psi}^2} + \sigma_{y}^2 + Z_{n\pm}^2 \sigma_{y'}^2 \right)}{\sqrt{\left( \sigma_x^2 + Z_{n\pm}^2 \sigma_{x'}^2 \right) \left( \frac{\varepsilon_y^2}{\sigma_{\psi}^2} + \sigma_{y}^2 + Z_{n\pm}^2 \sigma_{y'}^2 \right)}}
\]

\[
Z_{n\pm} = \lambda_w \left( n\pm \frac{1}{4} \right)
\]

and \( x_o = \frac{k \lambda_w}{\gamma 2\pi} \)

where \( \lambda_w \) is the wiggler period. Therefore, the calculation of the brightness for wiggler radiation must take into account the depth-of-field-effect-the contribution to the apparent source size from different poles.

The critical energy and total power of wiggler is

\[
E_c [keV] = 0.665 E^2 [GeV] B[T]
\]

\[
P_{tot} [kW] = 0.633 \cdot E^2 [GeV] B_o^2 [T] L [m] I [A]
\]
Undulator

Brightness and polarization rate: \[ \beta_n = \frac{B_n}{(2\pi)^2 \sigma_{TX} \sigma_{TY} \sigma_{TX} \sigma_{TY}} \]

The angular distribution of the nth harmonic is concentrated in a narrow cone whose half-width is given by

\[ \sigma_{\nu} = \sqrt{\frac{\lambda_n}{L}} = \frac{1}{\gamma} \sqrt{\frac{1 + \kappa^2}{2Nn}} \]

radiation photon energy \[ \varepsilon_n \text{[keV]} = \frac{0.95E^2 \text{[GeV]} \cdot n}{\lambda_n \left( 1 + \kappa^2 + \gamma^2 \left( \theta^2 + \psi^2 \right) \right)} \]

The total spectral flux \( B_n \) (Photons/s/mm²/mrad²/0.1%BW) and brilliance \( \beta_n \) (Photons/s/mm²/mrad²/0.1%BW) generated on the n-odd harmonic on-axis. \( \theta \) and \( \Psi \) are observation angle in the horizontal and vertical plane. The radiation by a single electron in the \( n \)th harmonic spectrum is characterized by the four Stokes parameter \( S = (S_0(n), S_1(n), S_2(n), S_3(n)) \).

\[ B_n = 1.431 \times 10^{14} nNI \left( \frac{S_0(n)}{1 + \frac{1}{2} \left( K_x^2 + K_y^2 \right)} \right) \]

Where

\[ S_0 \; \square \; n \; \square = V^2 + H_{\parallel}^2 + H_{\perp}^2 \]
\[ S_1 \; \square \; n \; \square = -V^2 + H_{\parallel}^2 + H_{\perp}^2 \]
\[ S_2 \; \square \; n \; \square = 2V H_{\parallel} \]
\[ S_3 \; \square \; n \; \square = 2V H_{\perp} \]
Undulator

With

\[ V = K_y J, \quad H_{\parallel} = K_x J \cos \phi, \quad H_\perp = K_x J \sin \phi. \]

And

\[ J = J_{(n+1)/2}(nK) - J_{(n-1)/2}(nK), \]

\[ D = \sqrt{K_x^2 + K_y^2 + 2K_x K_y \cos \phi} \]

\[ \frac{1}{4 \left( 1 + \frac{1}{2} (K_x^2 + K_y^2) \right)} \]

Where \( J_{n}(nK) \) represents the \( n \)-th integer order Bessel function as a function of the variable \( nK \), \( \phi \) is the phase difference between horizontal and vertical sinusoidal field.

- The r.m.s. Photon beam sizes and divergences are approximately given by

\[ \sigma_{T_x} = \sqrt{\sigma_p^2 + \sigma_x^2} \quad \sigma_{T_y} = \sqrt{\sigma_p^2 + \sigma_y^2} \]

\[ \sigma_{Tx'} = \sqrt{\sigma_{p'}^2 + \sigma_{x'}^2} \quad \sigma_{Ty'} = \sqrt{\sigma_{p'}^2 + \sigma_{y'}^2} \]

Where \( \sigma_x (\sigma_y) \) and \( \sigma_{x'} (\sigma_{y'}) \) represent the electron horizontal (vertical) beam sizes and divergences,

\[ \sigma_{p'} = \frac{\lambda}{L} = \sqrt{\frac{1 + (K_x^2 + K_y^2)/2}{2nN\gamma^2}} \quad \sigma_p = \frac{1}{4\pi} \sqrt{\frac{\lambda L}{\gamma}} \]
Undulator

- In helical undulator, the $n$-th harmonic energy of the helical radiation spectrum on-axis is

$$E_n[KeV] = \frac{0.95 \cdot n \cdot E^2[GeV]}{\lambda_u[cm]} \left( 1 + \frac{1}{2} (K_x^2 + K_y^2) + \gamma^2 (\theta^2 + \psi^2) \right)$$

- The polarization rates $P_i(n)$ of radiation are defined as the difference of the flux polarized along two orthogonal directions divided by the total polarization flux $P_i(n) = S_i(n)/S_0(n), i=1,2,3$. If the radiation is fully polarized one will find that

$$p = \sqrt{\sum_i p_i^2} = 1$$

- The criterion of the optimization between the flux and polarization rate on different phasing position should depend on the maximum value of $S_3^2 F_n (S_2^2 F_n)$ to define the merit flux.

Total power

$$P_{tot}[kW] = 0.633 \cdot E^2[GeV] B_o^2[T] L[m] I[A]$$

- Bandwidth $\Delta \lambda/\lambda = 1/Nn$, $N$ and $n$ are the periodic number and the $n$th harmonic number.
Features of elliptically polarized undulator undulator

Circular polarization n=1, higher harmonic spectrum is zero

Elliptical polarization for fundamental and higher harmonic spectrum
Spectral calculation

Flux

Brilliance

Photon Energy (KeV)

Photon Energy (KeV)
Field, first & second field integral
Power density calculation (total 5.96 kW)

Away from source 0.7 m
# Spectra calculation code

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SRW (ESFR):</strong></td>
<td></td>
</tr>
<tr>
<td>* User friendly package;</td>
<td>* Training course needed for familiarization;</td>
</tr>
<tr>
<td>* Associated with slit for beam line design;</td>
<td>* Documentation is not clear;</td>
</tr>
<tr>
<td>* Easy to do data process and data analysis;</td>
<td>* Large computer needed;</td>
</tr>
<tr>
<td>* Calculation spectrum &amp; power distribution;</td>
<td>* Program is not yet completed;</td>
</tr>
<tr>
<td>* For simple field calculation;</td>
<td>* Some parameters are not included;</td>
</tr>
<tr>
<td>* Fast calculation for FFT analysis spectrum</td>
<td>* Can down load from ESRF website</td>
</tr>
<tr>
<td>* Run in PC</td>
<td></td>
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<tr>
<td><strong>Spectra (SPing8):</strong></td>
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<tr>
<td>* User friendly package</td>
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</tr>
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<td>* Calculation spectrum &amp; power distribution</td>
<td>* Large use of memory;</td>
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<tr>
<td>* Easy to put parameters and data process</td>
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<td>* Taking into account different bata function</td>
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<td>* Run in PC</td>
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# Magnet Computation Codes for Magnet Design

<table>
<thead>
<tr>
<th>Advantages</th>
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<tbody>
<tr>
<td><strong>TOSCA:</strong></td>
<td>* Training course needed for familiarization;</td>
</tr>
<tr>
<td>* Full three dimensional package;</td>
<td>* Expensive to purchase;</td>
</tr>
<tr>
<td>* Accurate prediction of distribution and strength in 3D;</td>
<td>* Large computer needed.</td>
</tr>
<tr>
<td>* Extensive pre/post-processing;</td>
<td>* Large use of memory.</td>
</tr>
<tr>
<td>* Multipole function and Fast calculation</td>
<td>* Cpu time is hours for non-linear 3D problem.</td>
</tr>
<tr>
<td>* For static &amp; DC &amp; AC field calculation</td>
<td>* It can be run combined field</td>
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<tr>
<td>* Run in PC or workstation</td>
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**RADIA:**

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<td>* Large use of memory</td>
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<tr>
<td>* With quick-time to view and rotate 3D structure</td>
<td>* Be careful to make segmentation</td>
</tr>
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Design criteria of IDs

- **Wedged-poles** were shaped with a thicker cross section at pole tip.
- **Chamfers** are used to reduce local saturation and demagnetizing field.
- **Vertical recess** to minimize on-axis field strength variation.
- **Magnet overhang** reduces 3-D leakage flux and roll-off is slower.
- **Different thickness of magnet block sizes** with partial strength on the both end poles.
- **0.5 mm thickness shim** at magnet edge increase vertical field roll-off.
- **Two rows of trim magnets** for $B_y$ and $B_x$ multipole field shimming.
- **Magnet shim pieces** for trajectory and spectrum phase shimming.
- **Longitudinal distance** between each end pole, the **pole height**, and **pole tilt** can be adjustable.
Spectrum (phase) Shimming methods

\[ \Theta(z) = \frac{2\pi}{\lambda} \left( \frac{z}{2\gamma^2} - \int x'z^2 \, dz \right) \]

where \( x' = \frac{dx}{dz} \) represents the electron angle with respect to the undulator z-axis, \( \lambda \) is the photon radiation fundamental wavelength, and \( \gamma \) denotes the relativistic velocity. In the ideal undulator device, the phase at each pole should be a perfect linear variation and the phase error is zero.

However for a real undulator, the phase error \( \Delta\Theta \) is not zero and can be obtained by subtracting the two optimum linear fits of the real and ideal field

\[ I = I_0 e^{-\left(\frac{n\Delta\Theta_{rms}}{2}\right)^2} \]

Where \( I \) and \( I_0 \) represent the spectrum flux intensities with and without phase error.
Field quality control

To maintain the photon spectrum quality of the magnetic field measurement as good as the ideal magnetic field simulation, the deviation of $\Delta B_p / B_p$ between each pole peak field strength and the deviation of $\Delta I_{1/2} / I_{1/2}$ between the half period integral strength should be kept within 0.5%.

$$
\Delta B_p \equiv \frac{\sqrt{\sum_{i=1}^{i=N_p} \left( B_{\text{peak}} \right) - \left( \left| B_{\text{peak}} \right| \right)}^2}{B_p \left( \left| B_{\text{peak}} \right| \right)}.
$$

Where $B_{\text{peak}}$ and $I_{1/2}(\text{period}/2)$ denote the peak field strength and the half period integral field strength. These two factors determine the phase error $\Delta \phi_{\text{rms}}$

$$
\langle |B_{\text{peak}}| \rangle = \frac{1}{N_p} \sum_{i=1}^{i=N_p} |B_{\text{peak}}|.
$$

$$
\Delta I_{1/2} \equiv \frac{\sqrt{\sum_{i=1}^{i=N_p} \left( |I_{1/2}(\text{period}/2)| - \left( \left| I_{1/2}(\text{period}/2) \right| \right) \right)^2}}{I_{1/2} \left( \left| I_{1/2}(\text{period}/2) \right| \right)}.
$$

$$
\langle |I_{1/2}(\text{period}/2)| \rangle = \frac{1}{N_p} \sum_{i=1}^{i=N_p} |I_{1/2}(\text{period}/2)|.
$$
The shimming method has been studied to re-enlarge the dynamic aperture with the addition of a multipole field component. Such shims are placed on each of the four magnet arrays. They are designed based on the criteria of correcting the tune shift vs. x.
Multipole & spectrum shimming method

- Measuring the individual permanent magnet block and then arranging them by sorting block in the structure.
- Measuring the integral field strength of each block which on the keeper to reduce the mechanical error.
- Swapping blocks after assembly and field measurement.
- Using the thin iron pieces or permanent magnet pieces on magnet to correct the multipole and spectrum shimming.

Method of magnetic shimming to improve the magnetic field quality
Field quality control by various methods

\[ \int_{-\infty}^{\infty} (B_x + i B_y) dz \approx \sum_{n=0}^{n} (b_n + i a_n)(x+iy)^n. \]

Where \( a_n \) and \( b_n \) denote the integral normal and skew components.
Quasi-period undulator mechanism

The magnet structure of the quasi-periodic undulator was determined as $Z_n = n + (1/\eta - 1)[n/(\eta + 1) + 1]$, where $n$ is an integer, and $Z_n$ is the longitudinal coordinate which represents the $n$-th position of the magnet array. The $\eta$ is an irrational number, and the bracket $[]$ represents the maximum integer less than the number in the bracket. We select $\eta = 5^{1/2}$. 
Quasi-period undulator features

By Magnetic Field (T)

Displacement (um)

Flux Ratio (%) vs. Photon Energy (eV)

Photon Energy Shift Ratio (%) vs. Photon Energy (eV)
Hybrid magnet

Alternative hybrid designs; left - with wedged poles, right - with side magnets

Hybrid insertion device field strength enhancement techniques: (a) pole packing with PM material, (b) pole tapering.
Peak field strength on pure and hybrid magnet

Attainable on-axis field in pure PM and hybrid insertion devices ($B_r = 1.1$ T, $H_{pm} = -0.5940$ T)
Sequence of magnet poles (dotted line) resulting in no offset between the electron trajectory (solid line) and the magnet axis.

Various end-sequences for the pure-permanent magnet structure
Various planar permanent magnet arrangements. a – HELIOS, b – planar helical undulator, c – APPLE-II, d – Spring-8
Various schemes for introducing additional focusing in pure permanent magnet and hybrid undulators
EPU structure

ESRF’s planar device yields elliptically polarized light, yet does not impinge on the vacuum chamber.

Trieste’s proposed planar device produces gap-independent, circularly polarized light (handedness is invariant).

Block orientations for JAERI’s proposed APPLE-II planar device and SSRL’s elliptically polarizing undulator, and arbitrarily orientated linear polarization.
Elliptical polarization method

Asymmetric hybrid structure generates broadband elliptically polarized light.

Pair of crossed undulators separated by a tunable EM phase shifter.
Helical undulator

Millimeter-period dipole microundulator immersed in chain of alternating-polarity background fields.

Conductor configurations producing planar field in pulsed microundulators.

Superconducting In-vacuo undulator proposal by ANKA

R. Rossmanith
Special magnet

Solenoid-derived wiggler: copper-plated staggered poles deflect the B-field and simultaneously support microwaves.

Coaxial hybrid iron microundulator for annular beams.

Superconducting solenoid device-Helical stagger condulator
Helical undulator

A bifilar helical undulator produces circularly polarized light.

Two planar IDs rotated 90° produce arbitrary elliptical or arbitrarily oriented linear polarization.
Superconducting planar undulator with vertical wound racetrack coil
Superconducting helical undulator

Superconducting elliptical undulator structure

Superconducting undulator with variable polarization
Concept of superconducting helical undulator

\[
B_z = B_{zo} \cosh(\phi_z z) \cosh(\phi_x x) \cos(\phi_y y), \\
B_x = -\frac{\phi_x}{\phi_z} B_{zo} \sinh(\phi_z z) \sinh(\phi_x x) \cos(\phi_y y), \\
B_y = -\frac{\phi_y}{\phi_z} B_{zo} \sinh(\phi_z z) \cosh(\phi_x x) \sin(\phi_y y).
\]

\[
\phi_x^2 + \phi_z^2 = \phi_y^2 = (2\pi/\lambda_u)^2
\]

\[
B_z(x = z = 0, y) = B_{zo} \cos(2\pi y / \lambda_u), \\
B_x(x = z = 0, y) = -\frac{\phi_y}{\phi_z} B_{zo} \sin(2\pi y / \lambda_u) \sin \phi.
\]
Racetrack coil and staggered undulator with and without rotation

Staggered undulator

\[ B_z = -B_{y0} \cosh(\phi_z z) \cosh(\phi_x x) \cos(\phi_y y), \]
\[ B_x = -B_{y0} \sinh(\phi_z z) \sinh(\phi_x x) \cos(\phi_y y), \]
\[ B_y = B_0 - B_{y0} \cosh(\phi_z z) \cosh(\phi_x x) \sin(\phi_y y). \]
\[ B_{z0} = \frac{2B_0 \sin(\pi f)}{\sinh(\pi g / \lambda_u) \pi f} \]

\[ B_z(y) = -B_{y0} \cos(2\pi y / \lambda_u), \]
\[ B_x(y) = -B_{y0} \sin(2\pi y / \lambda_u) \sin \theta. \]
NSRRC U10 undulator
3 m long wiggler W20
SRRC U9 undulator
NSRRC EPU5.6 install in the storage ring
References

4rd OCPA Accelerator School

Insertion Devices-2

Wednesday 2 August, 2006 in YangZhou, 14:00–16:00

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Outlines

- Introduction
- Magnet field measurement method and system for insertion devices
- Field quality control procedure
- Magnet field measurement and analysis
- Example of insertion device measurement (NSRRC)
Measurement methods

- Flip and stretch wire system
- Three-orthogonal axes Hall probe system
- Highly automatic Helmholtz system
- Pulse-wire system
Stretch wire measurement system
3-D on fly Hall probe measurement system
Helmholtz coil system for magnet block measurement
On fly Hall probe measurement system and stretch wire measurement system
Hall probe measurement system to measure U10
3D-Hall probes on the fly mapping method

- **Advantages:**
  - Perform the point and integral field measurement for *only the straight magnets*
  - provide the criterion for the fast shimming of the photon spectrum, multipole components, electron trajectory of insertion devices
  - time consuming but the Measurement speed is much faster than the “on static” fixed angle Hall probe method.
  - *Real field measurement* of three magnetic field components without any field correction.

- **Crucial issues:**
  - Position calibration on a reference magnet should be very careful
  - The planar Hall effect should be considered
  - Field calibration should consider the orthogonal between the three Hall probe on the exact angle
Helmholtz coil method with 3-D rotation mechanism

- **Advantages:**
  - It is highly automatic and high speed measurement system for reducing the human error and the time consuming
  - The three *magnetic dipole moment components* can be obtained for once block installation
  - *Fast speed measurement for 90 s/magnet*

- **Crucial issues:**
  - It is a very complex system on the drive train mechanism
  - We should pre-run the system before starting the field measurement
  - Mechanical drive train system is not so reliable for a long time test
Stretch wire measurement system

- Advantages:
  - A simple mechanical structure with reliable high precision
  - Easy to exchange the measurement method and easy operation
  - It can be for first and second field integral measurement on insertion device and for harmonic field measurement on lattice magnets

- Crucial issues:
  - Electronic drift and gain in linearity of the integrator should be improved
  - The inexact coupling between the coupler and the rotor should be reduced
  - The deviation of wire rotating center and the sag of wire should be avoided
  - The wire length for the harmonic components measurement should be within 1 m long for Cu wire and 2.5 m long for Be-Cu wire to keep high precision of 0.01%
  - Use the Litz wire to enhance the resolution and accuracy
Pulse wire method

- Advantages and critical issues:
  - Make *in situ measurement* of wiggler magnetic field so as to monitor the field error and cancel the wiggler steering errors
  - In the point measurement of *mini-gap undulator*
  - To measure the *dynamic behavior* of the Pulsed magnets and current ramping of wiggler
  - To *speed the fine-tuning* of wiggler field
  - The wire imperfection from uniformity or impurity will create the wave distortion - improvement by thick wire
  - The dispersion of acoustic wave due to the phase variation can be reduced by increase the wire tension
  - Using the FFT and inverse FFT to perform the trace back procedure in order to compensate the dispersive wave
Stretch-wire System for Insertion Device Measurements
Stretch-wire System for Lattice Magnet Measurements

![Diagram of stretch-wire system for lattice magnet measurements](image)
Stretch-wire System for Integral Magnetic Field Measurements

First field integral measurement

\[ I_y = \frac{\int Vdt}{N \Delta x}, \quad I_x = \frac{\int Vdt}{N \Delta y} \]

Second field integral measurement

\[ II_y = -\frac{L}{2} \left[ \frac{\int Vdt}{N \Delta x} + I_y \right], \quad II_x = -\frac{L}{2} \left[ \frac{\int Vdt}{N \Delta y} + I_x \right] \]

Harmonic field components measurement

\[ \int_{\theta_i}^{\theta_{i+1}} Vdt = N \int_{z_L}^{z_0} \int B_r R d\theta \]

\[ \sum_{n=1}^{\infty} a_n \cos(n \theta) + b_n \sin(n \theta) = \sum_{n=1}^{\infty} 2 \frac{R^n N}{n} \left[ A_n \cos(n \theta) + B_n \sin(n \theta) \right] \sin \left( \frac{n \Delta \theta}{2} \right) \]

\[ A_{n, j}^{\text{skew}} = \frac{r_{\text{ref}}^{n-j} A_n}{B_{\text{ref}, j}}, \quad B_{n, j}^{\text{nor}} = \frac{r_{\text{ref}}^{n-j} B_n}{B_{\text{ref}, j}} \]
Stretch-wire System performance
Transverse x-axis (cm)

∫ B xds (G-cm)

∫ B xds of static scan
∫ B xds of dynamic scan
∫ B xds (G-cm)

(b) θ=90°

Flip

Translation

y=x=0

coil

(a) θ=θ

Translation

y=x=0

coil
\[ \delta = \frac{1}{2} \frac{\rho g}{T} z(z - L) \]

For Cu-Be

\[ \delta = \frac{L^2}{8} \left( \frac{9.47 \times 10^{-3} \times 1.1309 \times 10^{-4} \times 9.8}{12000 \times 1.1309 \times 10^{-4} \times 9.8} \right) \]

\[ = 0.987 \times 10^{-7} L^2 \text{ cm} \]

For Cu

\[ \delta = \frac{L^2}{8} \left( \frac{8.93 \times 10^{-3} \times 1.1309 \times 10^{-4} \times 9.8}{2500 \times 1.1309 \times 10^{-4} \times 9.8} \right) \]

\[ = 4.465 \times 10^{-7} L^2 \text{ cm} \]
Rotation coil
Helmholtz coil measurement method

\[
\vec{m} = m_x \hat{x} + m_y \hat{y} + m_z \hat{z},
\]
\[
m = \sqrt{m_x^2 + m_y^2 + m_z^2},
\]
\[
\alpha_x = \tan^{-1}\left(\frac{m_x}{m_z}\right),
\]
\[
\alpha_y = \tan^{-1}\left(\frac{m_y}{m_z}\right).
\]

\[
M_k = \vec{m} \cdot \hat{k} = m_x \hat{x} \cdot \hat{k} + m_y \hat{y} \cdot \hat{k} + m_z \hat{z} \cdot \hat{k}
= m_x \cos \theta_{xi,k} + m_y \cos \theta_{yi,k} + m_z \cos \theta_{zi,k},
\]
Pulse Wire Method

A comparison between thick pulse wire without trace back compensation and Hall probe measurement
The transverse-motion equation is first derived by Lord Rayleigh

\[-EI \frac{\partial^4 x}{\partial z^4} + T \frac{\partial^2 x}{\partial z^2} = \rho A \frac{\partial^2 x}{\partial t^2}\]

\(I\): moment of inertia
\(A\): cross section
\(E\): modulus of elasticity
\(\rho\): mass density
\(T\): tension

By considering the solution

\[x = Ce^{i(kz - \omega t)}\]

gives dispersion function

\[k(\omega) = \pm \sqrt{\frac{T}{2EI}} \sqrt{-1 \pm \sqrt{1 + 4 \frac{\rho AEI}{T^2} \omega^2}}\]

We have the following parameters,
\(d\): wire diameter = \(2.5 \times 10^{-4}\) m
\(I\): moment of inertia m\(^4\)
\(A\): cross section = \(4.909 \times 10^{-8}\) m\(^2\)
\(E\): modulus of elasticity = \(1.6 \times 10^{11}\) Pa
\(\rho\): mass density = 8250 kg/m\(^3\)
\(T\): tension=16N

Then we have the dispersion function

\[k(f) = 510 \sqrt{-1 \pm \sqrt{1 + 7.659 \times 10^{-9} f^2}}\]
The adjustment by trace-back process

1st Integral Field [Volt]

Z position [m]

Time [sec]

1st Integral Field [Gauss-cm]
A comparison between trace-back thick pulse wire calculation and Hall probe measurement

2nd Integral Field Measurement of Pulse Wire Method

Reduction of amplitude
Due to step-back
Tail signal

Wire diameter = 0.250mm
Pulse Duration = 1/\text{freq.} \times \text{duty cycle} = 100 \text{Hz} \times 1\% = 0.1 \text{ms}
Pulse amplitude from fun. generator = 4V, offset = 2V
Burst count = 1, burst period = 5 sec
Power supply of transistor, \( V_C = 20 \text{ V} \)
Oscilloscope vertical range = 300 mV
Oscilloscope sampling time = 20 µsec
AC sampling sweep number: 1624 pt
Average: 36, tension = 636+528+476=1640g

1st Integral Field Measurement

Optical signal [Volt]
Field quality control by various methods

- Measuring the individual permanent magnet block and then arranging them by sorting block in the structure.
- Measuring the integral field strength of each block which on the keeper to reduce the mechanical error.
- Swapping blocks after assembly and field measurement.
- Using the thin iron pieces or permanent magnet pieces on magnet to correct the multipole and spectrum shimming.

Method of magnetic shimming to improve the magnetic field quality
Schematic drawing of the magnet design of the EPU5.6 undulator
EPU performance

(a) Before re-alignment

(b) After re-alignment

Phase @ $p=0.64\pi$

Phase @ $p=-0.64\pi$

$\Delta B/B$ (%) vs Pole numbers
EPU performance

- Flux density (ph/s/mrad²/0.2A/0.1%BW)
- Flux ratio
- Polarization rate

Energy (keV)

Phase = 0

Phase = -\pi

Phase = 0.35\pi

Phase = -0.35\pi
3-D dimensional Hall probe and long-loop-flip coil measurement system
U9 performance

![Graph showing U9 performance with energy on the x-axis and ratio (%) on the y-axis. The graph includes lines for different values of n: n=1, n=3, n=5, n=7, n=9, and n=11.](image-url)
U10 performance

Trajectory (µm) vs. zaxis (m)

Gap = 23 mm

Angle (rad) vs. zaxis (m)

Gap = 23 mm
U10 performance
U10 performance

[Graphs showing the magnetic field components (Ix and Iy) vs. position (cm) with a gap of 23mm.]